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Quantum theory of electronic double-slit diffraction

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The phenomena of electron, neutron, atomic and molecular diffraction have been studied by many experiments, and these experiments are explained by some theoretical works. In this paper, we study electronic double-slit diffraction with quantum mechanical approach. We can obtain the results: (1) When the slit width a is in the range of $3\lambda \sim 50\lambda$ we can obtain the obvious diffraction patterns. (2) when the ratio of $\frac{d+a}{a} = n(n = 1, 2, 3, \cdots)$, order $2n, 3n, 4n, \cdots$ are missing in diffraction pattern. (3)When the ratio of $\frac{d+a}{a} \neq n(n = 1, 2, 3, \cdots)$, there isn't missing order in diffraction pattern. (4) We also find a new quantum mechanics effect that the slit thickness c has a large affect to the electronic diffraction patterns. We think all the predictions in our work can be tested by the electronic double-slit diffraction experiment.

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1. Introduction

The wave nature of subatomic particle elections and neutrons was postulated by de Broglie in 1923 and this idea can explain many diffraction experiments. The matter-wave diffraction has become a large field of interest throughout the last year, and it is extend to atom, more massive, complex objects, like large molecules I_2, C_{60} and C_{70}, which were found in experiments[12345]. In such experiments, the incoming atoms or molecules usually can be described by plane wave. As well known, the classical optics with its standard wave-theoretical methods and approximations, in particular those of Huygens and Kirchhoff, has been successfully applied to classical optics, and has yielded good agreement with many experiments. This simple wave-optical approach gives a description of matter wave diffraction also[67]. However, matter-wave interference and diffraction are quantum phenomena, and its fully description needs quantum mechanical approach. In this work, we study the double-slit diffraction of electron with quantum mechanical approach. In the view of quantum mechanics, the electron has the nature of wave, and the wave is described by wave function $\psi(\vec{r}, t)$, which can be calculated with Schrödinger's wave equation. The wave function $\psi(\vec{r}, t)$ has statistical meaning, i.e., $\mid \psi(\vec{r}, t) \mid^2$ can be explained as the particle's probability density at the definite position. For double-slit diffraction, if we can calculate the electronic wave function $\psi(\vec{r}, t)$ distributing on display screen, then we can obtain the diffraction intensity for the double-slit, since the diffraction intensity is directly proportional to $\mid \psi(\vec{r}, t) \mid^2$. In the double-slit diffraction, the electron wave functions can be divided into three areas. The first area is the incoming area, the electronic wave function is a plane wave. The second area is the double-slit area, where the electronic wave function can be calculated by Schrödinger's wave equation. The third area is the diffraction area, where the electronic wave function can be obtained by the Kirchhoff's law. In the following, we will calculate these wave functions.

The paper is organized as follows. In section 2 we calculate the electronic wave function in the double-slit with quantum mechanical approach. In section 3 we calculate the electronic wave function in diffraction area with the Kirchhoff's law. Section 4 is numerical analysis, Section 5 is a summary of results and conclusion.

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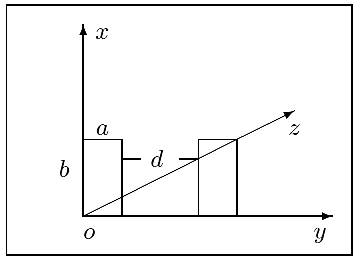

FIG. 1: The double-slit geometry, the width a, the length b and the two slit distance d.

2. Quantum theory of electron diffraction in double-slit

In an infinite plane, we consider a double-slit, its width a, length b and the two slit distance d are shown in FIG. 1. The x axis is along the slit length b and the y axis is along the slit width a. In the following, we calculate the electron wave function in the first single-slit (left slit) with Schrodinger equation, and the electron wave function of the second single-slit(right slit) can be obtained easily. At a time t, we suppose that the incoming plane wave travels along the z axis. It is

$$\Psi_1(z,t) = Ae^{\frac{i}{\hbar}(pz-Et)}, \tag{1}$$

where A is a constant.

The potential in the single-slit is

$$\begin{aligned} V(x,y,z) &= 0 \qquad 0 \le x \le b, 0 \le y \le a, 0 \le z \le c \\ &= \infty \qquad otherwise, \end{aligned} \tag{2}$$

where c is the thickness of the single-slit. The time-dependent and time-independent Schrodinger equations are

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = -\frac{\hbar^2}{2M}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\psi(\vec{r},t), \tag{3}$$

$$\frac{\partial^2\psi(\vec{r})}{\partial x^2} + \frac{\partial^2\psi(\vec{r})}{\partial y^2} + \frac{\partial^2\psi(\vec{r})}{\partial z^2} + \frac{2ME}{\hbar^2}\psi(\vec{r}) = 0, \tag{4}$$

where $M(E)$ is the mass(energy) of the electron. In Eq. (4), the wave function $\psi(x,y,z)$ satisfies boundary conditions

$$\psi(0,y,z) = \psi(b,y,z) = 0, \tag{5}$$

$$\psi(x,0,z) = \psi(x,a,z) = 0, \tag{6}$$

The partial differential equation (4) can be solved by the method of separation of variable. By writing

$$\psi(x,y,z) = X(x)Y(y)Z(z), \tag{7}$$

Eq. (4) becomes

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} + \frac{2ME}{\hbar^2} = 0, \tag{8}$$

and Eq. (8) can be written into the following three equations

$$\frac{1}{X}\frac{d^2X}{dx^2} + \lambda_1 = 0, \tag{9}$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} + \lambda_2 = 0, \tag{10}$$

$$\frac{1}{Z}\frac{d^2Z}{dx^2}+\lambda_3=0, \tag{11}$$

where λ_1, λ_2 and λ_3 are constants,which satisfy

$$\frac{2ME}{\hbar^2}=\lambda_1+\lambda_2+\lambda_3. \tag{12}$$

From Eq. (5) and (6), with $X(x)$ and $Y(y)$ satisfying the boundary conditions

$$\begin{aligned} X(0)=X(b)=0 \\ Y(0)=Y(a)=0, \end{aligned} \tag{13}$$

we can obtain the equations of $X(x)$ and $Y(y)$

$$\begin{aligned} &\frac{d^2X}{dx^2}+\lambda_1X=0 \\ &X(0)=0 \\ &X(b)=0, \end{aligned} \tag{14}$$

and

$$\begin{aligned} &\frac{d^2Y}{dy^2}+\lambda_2Y=0 \\ &Y(0)=0 \\ &Y(a)=0, \end{aligned} \tag{15}$$

their eigenfunctions and eigenvalues are

$$\begin{aligned} &X_n(x)=A_n\sin\frac{n\pi}{b}x \qquad (n=1,2,\cdots) \\ &\lambda_1=(\frac{n\pi}{b})^2, \end{aligned} \tag{16}$$

and

$$\begin{aligned} &Y_m(y)=B_m\sin\frac{m\pi}{a}y \qquad (m=1,2,\cdots) \\ &\lambda_2=(\frac{m\pi}{a})^2. \end{aligned} \tag{17}$$

The solution of Eq. (11) is

$$Z_{mn}(z)=C_{mn}e^{i\sqrt{\frac{2ME}{\hbar^2}-\frac{n^2\pi^2}{b^2}-\frac{m^2\pi^2}{a^2}}z}, \tag{18}$$

and the particular solution of the wave equation (4) is

$$\begin{aligned} \psi_{mn} &= X_n(x)Y_m(y)Z_{mn}(z) \\ &= A_nB_mC_{mn}\sin\frac{n\pi x}{b}\sin\frac{m\pi y}{a}e^{i\sqrt{\frac{2ME}{\hbar^2}-\frac{n^2\pi^2}{b^2}-\frac{m^2\pi^2}{a^2}}z} \\ &= D_{mn}\sin\frac{n\pi x}{b}\sin\frac{m\pi y}{a}e^{i\sqrt{\frac{2ME}{\hbar^2}-\frac{n^2\pi^2}{b^2}-\frac{m^2\pi^2}{a^2}}z}. \end{aligned} \tag{19}$$

The time-dependent particular solution of Eq. (3) is

$$\psi_{mn}(x,y,z,t)=\psi_{mn}(x,y,z)e^{-\frac{i}{\hbar}Et}. \tag{20}$$

The general solution of Eq. (3) is

$$\begin{aligned} \psi_2(x,y,z,t) &= \sum_{mn}\psi_{mn}(x,y,z,t) \\ &= \sum_{mn}D_{mn}\sin\frac{n\pi x}{b}\sin\frac{m\pi y}{a}e^{i\sqrt{\frac{2ME}{\hbar^2}-\frac{n^2\pi^2}{b^2}-\frac{m^2\pi^2}{a^2}}z}e^{-\frac{i}{\hbar}Et}. \end{aligned} \tag{21}$$

Eq. (21) is the electronic wave function in the first single-slit. Since the wave functions are continuous at $z=0$, we have

$$\psi_1(x,y,z,t)\mid_{z=0}=\psi_2(x,y,z,t)\mid_{z=0}, \tag{22}$$

from Eq. (22), we obtain

$$\sum_{mn} D_{mn} \sin\frac{n\pi x}{b}\sin\frac{m\pi y}{a} = A, \tag{23}$$

where D_{mn} is a coefficient, which is

$$\begin{aligned} D_{mn} &= \frac{4}{ab}\int_0^a\int_0^b A\sin\frac{n\pi\xi}{b}\sin\frac{m\pi\eta}{a}d\xi d\eta \\ &= \frac{16A}{mn\pi^2} \qquad\qquad m, n, odd \\ &= 0 \qquad\qquad\qquad otherwise, \end{aligned} \tag{24}$$

substituting Eq. (24) into (21), we can obtain the electronic wave function in the first single-slit.

$$\begin{aligned} \psi_2(x,y,z,t) &= \sum_{m,n=0}^{\infty}\frac{16A}{(2m+1)(2n+1)\pi^2}\sin\frac{(2n+1)\pi x}{b}\sin\frac{(2m+1)\pi y}{a} \\ & e^{i\sqrt{\frac{2ME}{\hbar^2}-\frac{(2n+1)^2\pi^2}{b^2}-\frac{(2m+1)^2\pi^2}{a^2}}z}e^{-\frac{i}{\hbar}Et}. \end{aligned} \tag{25}$$

The electron wave function in the second single-slit can be obtained by making the coordinate translation:

$$\begin{aligned} x^{'} &= x \\ y^{'} &= y-(a+d) \\ z^{'} &= z, \end{aligned} \tag{26}$$

on substituting Eq. (26) into (25), we can obtain the electron wave function $\psi_3(x,y,z,t)$ in the second single-slit

$$\begin{aligned} \psi_3(x,y,z,t) &= \sum_{m,n=0}^{\infty}\frac{16A}{(2m+1)(2n+1)\pi^2}\sin\frac{(2n+1)\pi x}{b}\sin\frac{(2m+1)\pi(y-(a+d))}{a} \\ & e^{i\sqrt{\frac{2ME}{\hbar^2}-\frac{(2n+1)^2\pi^2}{b^2}-\frac{(2m+1)^2\pi^2}{a^2}}z}e^{-\frac{i}{\hbar}Et}. \end{aligned} \tag{27}$$

3. The wave function of electron diffraction

With the Kirchhoff's law, we can calculate the electron wave function in the diffraction area. It can be calculated by the formula[8]

$$\psi_{out}(\vec{r},t) = -\frac{1}{4\pi}\int_s\frac{e^{ikr}}{r}\overrightarrow{n}\cdot[\bigtriangledown^{'}\psi_{in}+(ik-\frac{1}{r})\frac{\overrightarrow{r}}{r}\psi_{in}]ds, \tag{28}$$

where $\psi_{out}(\vec{r},t)$ is diffraction wave function on display screen, $\psi_{in}(\vec{r},t)$ is the wave function of slit surface (z=c) and s is the area of the aperture or slit. The Kirchhoff formula (28) is approximate, since it neglects the effect of diffraction aperture or slit on the incoming wave $\psi_{in}(\vec{r},t)$. However, when the diffraction aperture or slit is larger than the electron wave length the effect can be neglected.
For the double-slit diffraction, Eq. (28) becomes

$$\begin{aligned} \psi_{out}(\vec{r},t) = & -\frac{1}{4\pi}\int_{s_1}\frac{e^{ikr}}{r}\overrightarrow{n}\cdot[\bigtriangledown^{'}\psi_2+(ik-\frac{1}{r})\frac{\overrightarrow{r}}{r}\psi_2]ds \\ & -\frac{1}{4\pi}\int_{s_2}\frac{e^{ikr}}{r}\overrightarrow{n}\cdot[\bigtriangledown^{'}\psi_3+(ik-\frac{1}{r})\frac{\overrightarrow{r}}{r}\psi_3]ds. \end{aligned} \tag{29}$$

In Eq. (29), the first and second terms are corresponding to the diffraction wave functions of the first slit and the second slit.

In the following, we firstly calculate the diffraction wave function of the first slit, it is

$$\psi_{out_1}(\vec{r},t) = -\frac{1}{4\pi}\int_{s_1}\frac{e^{ikr}}{r}\overrightarrow{n}\cdot[\bigtriangledown^{'}\psi_2+(ik-\frac{1}{r})\frac{\overrightarrow{r}}{r}\psi_2]ds, \tag{30}$$

The diffraction area is shown in FIG. 2, where $k=\sqrt{\frac{2ME}{\hbar^2}}$, s_1 is the area of the first single-slit, $\overrightarrow{r}^{'}$ is the position of a point on the surface (z=c), P is an arbitrary point in the diffraction area, and $\overrightarrow{n}$ is a unit vector, which is normal to the surface of the slit.

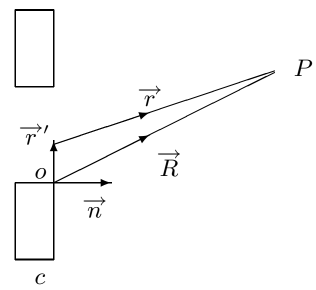

FIG. 2: The diffraction area of single-slit

From FIG. 2, we have

$$\begin{aligned} r &= R - \frac{\overrightarrow{R}}{R} \cdot \overrightarrow{r}' \\ &\approx R - \frac{\overrightarrow{r}}{r} \cdot \overrightarrow{r}' \\ &= R - \frac{\overrightarrow{k_2}}{k} \cdot \overrightarrow{r}', \end{aligned} \tag{31}$$

then,

$$\begin{aligned} \frac{e^{ikr}}{r} &= \frac{e^{ik(R-\frac{\overrightarrow{r}}{r}\cdot\overrightarrow{r}')}}{R-\frac{\overrightarrow{r}}{r}\cdot\overrightarrow{r}'} \\ &= \frac{e^{ikR}e^{-i\overrightarrow{k_2}\cdot\overrightarrow{r}'}}{R-\frac{\overrightarrow{r}}{r}\cdot\overrightarrow{r}'} \\ &\approx \frac{e^{ikR}e^{-i\overrightarrow{k_2}\cdot\overrightarrow{r}'}}{R} \qquad (|\overrightarrow{r}'| \ll R), \end{aligned} \tag{32}$$

with $\vec{K_2} = K\frac{\vec{r}}{r}$. Substituting Eq. (31) and (32) into (30), one can obtain

$$\psi_{out_1}(\vec{r},t) = -\frac{e^{ikR}}{4\pi R}\int_{s_0} e^{-i\overrightarrow{k_2}\cdot\overrightarrow{r}'}\overrightarrow{n}\cdot[\bigtriangledown' \psi_2(x',y',z') + (i\overrightarrow{k}_2 - \frac{\overrightarrow{R}}{R^2})\psi_2(x',y',z')]ds'. \tag{33}$$

In Eq. (33), the term

$$\begin{aligned} \overrightarrow{n}\cdot\nabla'\psi_2(x',y',z')|_{z=c} &= n_x\frac{\partial\psi_2(\overrightarrow{r}')}{\partial x'} + n_y\frac{\partial\psi_2(\overrightarrow{r}')}{\partial y'} + n_z\frac{\partial\psi_2(\overrightarrow{r}')}{\partial z'} \\ &= n_z\frac{\partial\psi_2(\overrightarrow{r}')}{\partial z'} \\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{16A}{(2m+1)(2n+1)\pi^2} \\ &\quad i\sqrt{\frac{2ME}{\hbar^2} - (\frac{(2n+1)\pi}{b})^2 - (\frac{(2m+1)\pi}{a})^2} \\ &\quad e^{i\sqrt{\frac{2ME}{\hbar^2} - (\frac{(2n+1)\pi}{b})^2 - (\frac{(2m+1)\pi}{a})^2}\cdot c} \\ &\quad \sin\frac{(2n+1)\pi}{b}x' \sin\frac{(2m+1)\pi}{a}y', \end{aligned} \tag{34}$$

then Eq. (33) is

$$\begin{aligned} \psi_{out_1}(\vec{r},t) &= -\frac{e^{ikR}}{4\pi R}e^{-\frac{i}{\hbar}Et}\int_{s_0} e^{-i\overrightarrow{k_2}\cdot\overrightarrow{r}'}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{16A}{(2m+1)(2n+1)\pi^2} \\ &\quad e^{i\sqrt{\frac{2ME}{\hbar^2} - (\frac{(2n+1)\pi}{b})^2 - (\frac{(2m+1)\pi}{a})^2}\cdot c}\sin\frac{(2n+1)\pi}{b}x'\sin\frac{(2m+1)\pi}{a}y' \\ &\quad [i\sqrt{\frac{2ME}{\hbar^2} - (\frac{(2n+1)\pi}{b})^2 - (\frac{(2m+1)\pi}{a})^2} + i\overrightarrow{n}\cdot\overrightarrow{k_2} - \frac{\overrightarrow{n}\cdot\overrightarrow{R}}{R^2}]dx'dy'. \end{aligned} \tag{35}$$

Assume that the angle between $\overrightarrow{k_2}$ and x axis (y axis) is $\frac{\pi}{2}-\alpha$ ($\frac{\pi}{2}-\beta$), and $\alpha(\beta)$ is the angle between $\overrightarrow{k_2}$ and the surface of yz (xz), then we have

$$k_{2x} = k\sin\alpha, \qquad k_{2y} = k\sin\beta, \tag{36}$$

$$\overrightarrow{n}\cdot\overrightarrow{k_2} = k\cos\theta, \tag{37}$$

where θ is the angle between $\overrightarrow{k_2}$ and z axis, and the angles θ, α, β satisfy the equation

$$\cos^2\theta + \cos^2(\frac{\pi}{2}-\alpha) + \cos^2(\frac{\pi}{2}-\beta) = 1. \tag{38}$$

Substituting Eq. (36) - (38) into (35) gives

$$\begin{aligned}\psi_{out_1}(x,y,z,t) &= -\frac{e^{ikR}}{4\pi R}e^{-\frac{i}{\hbar}Et}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{16A}{(2m+1)(2n+1)\pi^2}e^{i\sqrt{\frac{2ME}{\hbar^2}-(\frac{(2n+1)\pi}{b})^2-(\frac{(2m+1)\pi}{a})^2}\cdot c}\\ &\quad [i\sqrt{\frac{2ME}{\hbar^2}-(\frac{(2n+1)\pi}{b})^2-(\frac{(2m+1)\pi}{a})^2}+(ik-\frac{1}{R})\sqrt{\cos^2\alpha-\sin^2\beta}]\\ &\quad \int_0^b e^{-ik\sin\alpha\cdot x'}\sin\frac{(2n+1)\pi}{b}x'dx'\int_0^a e^{-ik\sin\beta\cdot y'}\sin\frac{(2m+1)\pi}{a}y'dy'.\end{aligned} \tag{39}$$

Eq. (39) is the diffraction wave function of the first slit. Obviously, the diffraction wave function of the second slit is

$$\begin{aligned}\psi_{out_2}\quad(x,y,z,t) &= -\frac{e^{ikR}}{4\pi R}e^{-\frac{i}{\hbar}Et}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{16A}{(2m+1)(2n+1)\pi^2}e^{i\sqrt{\frac{2ME}{\hbar^2}-(\frac{(2n+1)\pi}{b})^2-(\frac{(2m+1)\pi}{a})^2}\cdot c}\\ &\quad [i\sqrt{\frac{2ME}{\hbar^2}-(\frac{(2n+1)\pi}{b})^2-(\frac{(2m+1)\pi}{a})^2}+(ik-\frac{1}{R})\sqrt{\cos^2\alpha-\sin^2\beta}]\\ &\quad \int_0^b e^{-ik\sin\alpha\cdot x'}\sin\frac{(2n+1)\pi}{b}x'dx'\int_{a+d}^{2a+d} e^{-ik\sin\beta\cdot y'}\sin\frac{(2m+1)\pi}{a}(y'-(a+d))dy',\end{aligned} \tag{40}$$

where d is the two slit distance. The total diffraction wave function for the double-slit is

$$\psi_{out}(x,y,z,t) = \psi_{out_1}(x,y,z,t) + \psi_{out_2}(x,y,z,t) \tag{41}$$

From the diffraction wave function $\psi_{out}(x,y,z,t)$, we can obtain the relative diffraction intensity I on the display screen, it is

$$I \propto |\psi_{out}(x,y,z,t)|^2. \tag{42}$$

4. Numerical result

In this section we present our numerical calculation of relative diffraction intensity. The main input parameters are: $M = 9.11\times10^{-31}kg$, $R = 1m$, $A = 10^8$, $\alpha = 0.01rad$, $E = 0.001eV$ and the Planck constant $\hbar = 1.055\times10^{-34}Js$. We can obtain the relation between the diffraction angle β and relative diffraction intensity I. In double-slit diffraction, we can obtain the results: (1) When the ratio of $\frac{d+a}{a} = n(n = 1,2,3,\cdots)$, order $2n, 3n, 4n, \cdots$ are missing in diffraction pattern. (2)When the ratio of $\frac{d+a}{a} \neq n(n = 1,2,3,\cdots)$, there isn't missing order in diffraction pattern. In FIG. 3 and FIG. 4, we take $a = \lambda$, $b = 1000\lambda$ and $c = \lambda$, the diffraction patterns are not obvious. In FIG. 3, the ratio of $\frac{d+a}{a} = 6$, the order 6 is missing. In FIG. 4, the ratio of $\frac{d+a}{a} = 6.5$, there isn't missing order. In FIG. 5, FIG. 6 and FIG. 7, we take $a = \lambda, b = 1000\lambda, c = \lambda$, the diffraction patterns are obvious, where $\lambda = \frac{2\pi\hbar}{\sqrt{2ME}}$ is electronic wavelength. In FIG.5, the ratio of $\frac{d+a}{a} = 3$, the orders $3, 6, \cdots$ are missing. In FIG. 6, the ratio of $\frac{d+a}{a} = 3.4$, there isn't missing order. In FIG. 7, the ratio of $\frac{d+a}{a} = 6$, the orders $6, 12, \cdots$ are missing. In FIG. 8, FIG. 9 and FIG.10, the slit width a are corresponding to 20λ, 30λ and 50λ, their diffraction patterns are obvious. In FIG. 8, FIG. 9 and FIG. 10, the ratio of $\frac{d+a}{a} = 3$, the orders $3, 6, \cdots$ are missing. From FIG. 11 to FIG. 14, the slit thickness c is corresponding to 0, 10λ, 100λ and 1000λ. We can find that the thickness c can make a large impact on the double-slit diffraction pattern. When the slit thickness c increases the peak values of diffraction pattern increase also.

5. Conclusion

In conclusion, we studied the double-slit diffraction phenomenon of electron with quantum mechanical approach. We give the relation between diffraction angle and the relative diffraction intensity. We find the following results: (1) When the slit width a is in the range of $3\lambda \sim 50\lambda$ we can obtain the obvious diffraction patterns. (2) when the ratio of $\frac{d+a}{a} = n(n = 1, 2, 3, \cdots)$, order $2n, 3n, 4n, \cdots$ are missing in diffraction pattern. (3)When the ratio of $\frac{d+a}{a} \neq n(n = 1, 2, 3, \cdots)$, there isn't missing order in diffraction pattern. (4) We also find a new quantum mechanics effect that the slit thickness c has a large affect to the electronic diffraction patterns. We think all the predictions in our work can be tested by the electronic double-slit diffraction experiment.

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[1] O. Carnal and J. Mlynek, Phys. Rev. Lett. **66**, 2689 (1991).
[2] W. Schöllkopf, J. P. Toennies, Science **266**, 1345 (1994).
[3] M. Arudt, O. Nairz, J. Voss-Andreae, C. Kwller, G. Vander Zouw, and A. Zeilinger, Nature **401**, 680 (1999).
[4] O. Nairz, M. Arudt and A. Zeilinger, J. Mod. Opt. **47**, 2811 (2000).
[5] S. Kunze, K. Dieckmann and G. Rempe, Phys. Rev. Lett. **78**, 2038 (1997).
[6] B. Brezger, L. Hackermuller, S. Uttenthaler, J. Petschinka, M. Arndt, A. Zeilinger, Phys. Rev. Lett. **88**, 100404 (2002).
[7] A.S. Sanz, F. Borondo and M.J. Bastiaans, Phys. Rev. A **71**, 042103 (2005).
[8] M. Schwartz, Principles of Electrodynamics, Oxford University Press, 1972.

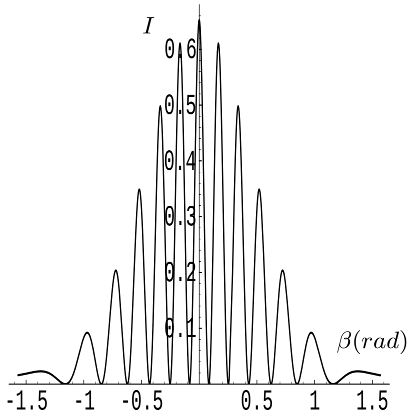

FIG. 3: The relation between β and I with $a=\lambda$, $b=1000\lambda$, $c=\lambda$ and $d=5\lambda$.

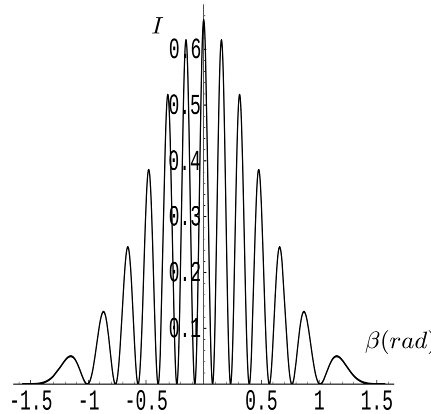

FIG. 4: The relation between β and I with $a=\lambda$, $b=1000\lambda$, $c=\lambda$ and $d=5.5\lambda$.

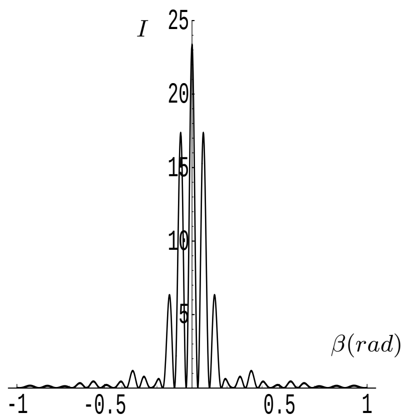

FIG. 5: The relation between β and I with $a=5\lambda$, $b=1000\lambda$, $c=\lambda$ and $d=10\lambda$.

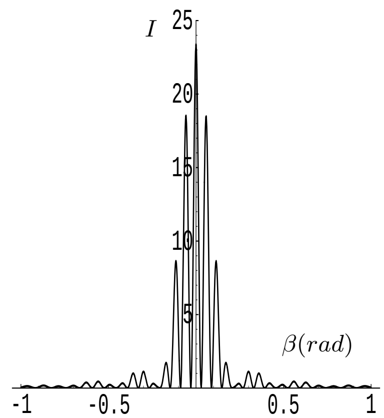

FIG. 6: The relation between β and I with $a=5\lambda$, $b=1000\lambda$, $c=\lambda$ and $d=12\lambda$.

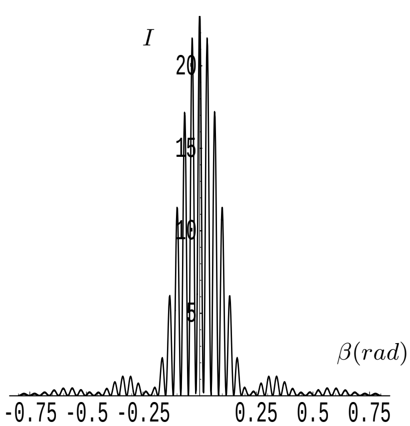

FIG. 7: The relation between β and I with $a=5\lambda$, $b=1000\lambda$, $c=\lambda$ and $d=25\lambda$.

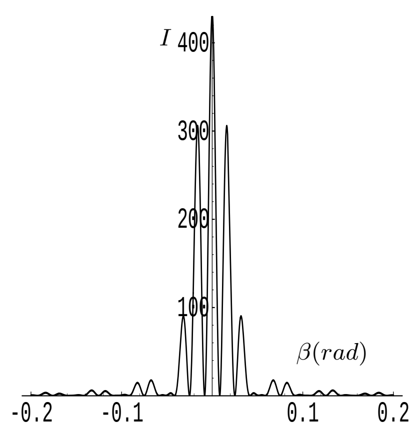

FIG. 8: The relation between β and I with $a=20\lambda$, $b=1000\lambda$, $c=\lambda$ and $d=40\lambda$.

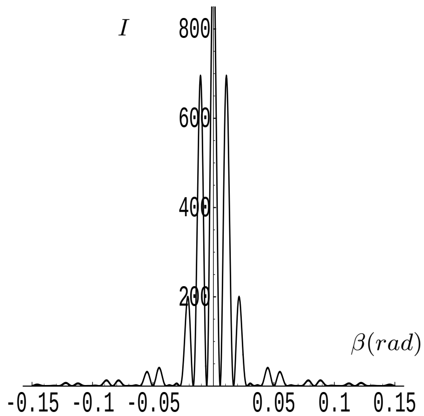

FIG. 9: The relation between β and I with $a = 30\lambda$, $b = 1000\lambda$, $c = \lambda$ and $d = 60\lambda$.

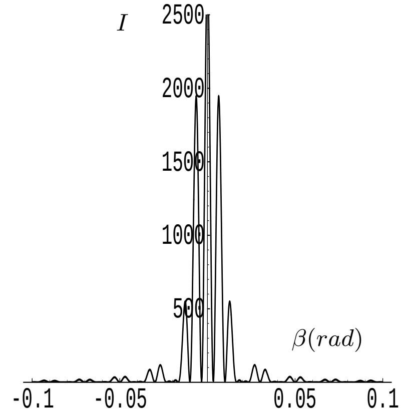

FIG. 10: The relation between β and I with $a = 50\lambda$, $b = 1000\lambda$, $c = \lambda$ and $d = 100\lambda$.

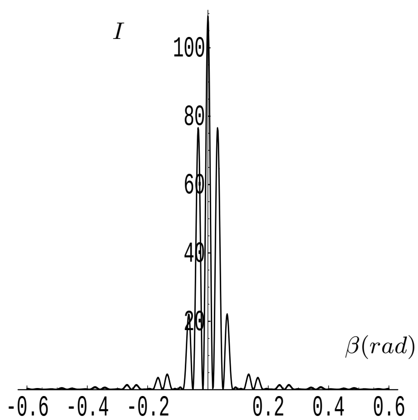

FIG. 11: The relation between β and I with $a = 10\lambda$, $b = 1000\lambda$, $c = 0$ and $d = 20\lambda$.

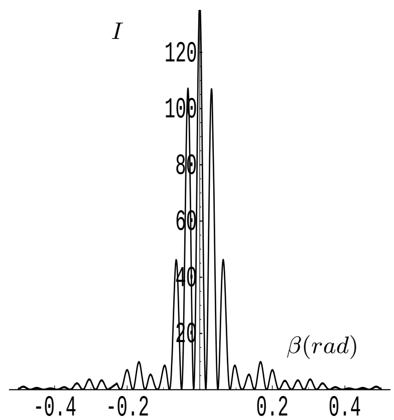

FIG. 12: The relation between β and I with $a = 10\lambda$, $b = 1000\lambda$, $c = 10\lambda$ and $d = 20\lambda$.

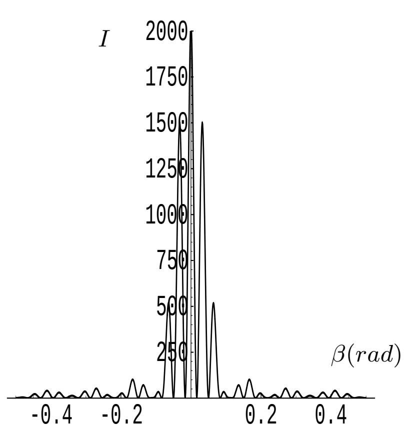

FIG. 13: The relation between β and I with $a = 10\lambda$, $b = 1000\lambda$, $c = 100\lambda$ and $d = 20\lambda$.

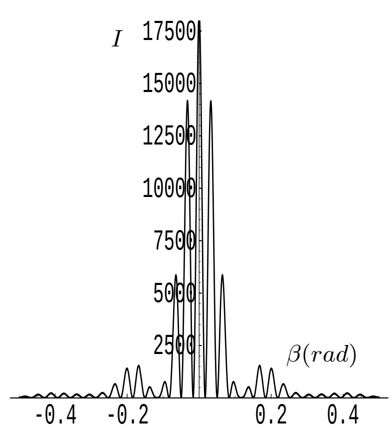

FIG. 14: The relation between β and I with $a = 10\lambda$, $b = 1000\lambda$, $c = 1000\lambda$ and $d = 20\lambda$.